NagpurStudy.com

(C) Show that $w = \dfrac{5-4z}{4z-2}$ transform the circle $|z| = 1$ into a circle of radius unity in w-plane and find the centre of the circle. 6

(D) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$ and $w_3 = -1$ respectively. 6

5. (A) For the function f(x) defined by $f(x) = 0, -\pi \le x < 0$ and $f(x) = \pi, 0 \le x < \pi$, show that the Fourier series converges to $\pi/2$ at the point of discontinuity $x = \pi$. 1½

(B) Find the Fourier coefficients a_0 and a_n for the function defined by $f(x) = 0$ for $-2 \le x < 0$ and $f(x) = 1$ for $0 \le x < 2$. 1½

(C) If $f_1(x) \le f_2(x)$ on [a, b], then prove that

$$\int_a^b f_1 \, d\alpha \le \int_a^b f_2 \, d\alpha.$$ 1½

(D) For any partition P of [a, b], prove that $L(P, f) \le U(P, f)$. 1½

(E) Prove that an analytic function with constant real part is constant. 1½

(F) Prove that the function $f(z) = xy + iy$ is not analytic. 1½

(G) Find the fixed points of the bilinear transformation

$$w = \frac{z}{z-2}.$$ 1½

(H) Show that $w = iz + i$ maps half plane $x > 0$ onto half plane $v > 1$. 1½

Bachelor of Science B.Sc. Semester–V (C.B.S) Examination

ANALYSIS

(M_9 Mathematics Paper–I)

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions 1 to 4 have an alterative; solve each question in full or its alternative in full.

UNIT—I

1. (A) If the Fourier series of the function f(x) on $-\pi \le x \le \pi$ is defined by

$$f(x) = \frac{a_0}{z} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx),$$ then find the Fourier coefficients a_0, a_n and b_n. 6

(B) Find the Fourier series of the function defined by $f(x) = 0, -\pi \le x < 0$ and $f(x) = \pi, 0 \le x \le \pi$. 6

OR

(C) If α is not an integer, then show that

$$\cos \alpha x = \frac{\sin \alpha\pi}{\alpha\pi} + \frac{2\alpha \sin \alpha\pi}{\pi} \sum_{1}^{\infty} \frac{(-1)^n \cos nx}{\alpha^2 - n^2}$$ for $-\pi \le x \le \pi$. 6

rtmnuonline.com

NAGPURSTUDY.COM

(D) Find the Sine series for the function $f(x) = \cos x,\ 0 \le x \le \pi$. 6

UNIT—II

2. (A) If P* is a refinement of a partition P of [a, b], then prove that :

$U(P^*, f, \alpha) \le U(P, f, \alpha)$.

where α is a monotonically increasing function on [a, b]. 6

(B) Let $f \in R(\alpha)$ on [a, b]. For $a \le x \le b$, put

$F(x) = \int_a^x f(t)\,dt$. Then prove that the F is continuous on [a, b]. 6

OR

(C) If $f \in R$ on [a, b] and if there is a differentiable function F on [a, b] such that F' = f, then prove that

$$\int_a^b f(x)\,dx = F(b) - F(a).$$ 6

(D) If f is contrinuous on [a, b] then prove that $f \in R(\alpha)$ on [a, b]. 6

UNIT—III

3. (A) If f(z) = u + iv is an analytic function and $z = re^{i\theta}$ where u, v, r, θ are all real, show that the Cauchy Reimann equations are :

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta},\ \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$ 6

(B) If f(z) is an analytic function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$ 6

OR

(C) If u and v are harmonic in a region R, then prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic in R. 6

(D) If $u = x^3 - 3xy^2$, show that there exists a function v(x, y) such that w = u + iv is analytic in a finite region. 6

UNIT—III

4. (A) Determine the region R' in w-plane corresponding to the triangular region R bounded by the lines x = 0, y = 0 and x + y = 1 in z-plane under the transformation $w = z\,e^{i\pi/4}$. 6

(B) If there are distinct invariant points p and q, then show that the bilinear transformation may be put in the form $\frac{w-p}{w-q} = k\frac{(z-p)}{(z-q)}$, where $k = \frac{a-cp}{a-cq}$. 6

OR

rtmnuonline.com